Cosmological nucleosynthesis and active-sterile neutrino oscillations with small mass differences: The resonant case

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Abstract

We have provided a numerical study of the influence of the resonant active-sterile neutrino oscillations $\nu_e \leftrightarrow \nu_s$, on the primordial production of helium-4. The evolution of the neutrino ensembles was followed selfconsistently with the evolution of the nucleons, using exact kinetic equations for the neutrino density matrix and the nucleon number densities in momentum space, from the time of neutrino decoupling till the freeze-out of nucleons at 0.3 MeV.

The exact kinetic approach enabled us to study precisely the neutrino depletion, spectrum distortion and neutrino mixing generated asymmetry due to oscillations at each momentum mode, and to prove that their effect on nucleosynthesis is considerable.

We have calculated the dependence of the primordially produced helium-4 on the oscillation parameters $Y_p(\delta m^2, \vartheta)$ for the full range of mixing parameters of the model of oscillations with small mass differences $\delta m^2 \leq 10^{-7}$ eV^2. We have obtained iso-helium contours on the $\delta m^2 - \vartheta$ plane. Cosmological constraints on oscillation parameters, more precise than the existing ones were extracted, due to the exact kinetic approach and the proper account for the neutrino spectrum distortion and the oscillations generated asymmetry.

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1 Overview of neutrino oscillations and primordial nucleosynthesis

Nowadays several major experiments point to the existence of neutrino oscillations. Besides, the neutrino puzzles, namely, the solar neutrino deficit and the atmospheric neutrino anomaly, which are believed to be explained in terms of neutrino oscillations, are still with us. Therefore, it seems useful to obtain more precise cosmological constraints on neutrino oscillation parameters. Moreover, the very low values of mass differences, explored in the cosmological model we will discuss, are beyond the reach of present and near future experimental constraints. In the present work we explore the effect of resonant active-sterile neutrino oscillations with small mass differences $\delta m^2 \leq 10^{-7}$ eV^2 on the primordial production of helium-4 and obtain precise cosmological constraints on the neutrino oscillation parameters.

The problem of neutrino oscillations and Big Bang Nucleosynthesis (BBN) has been discussed in numerous publications [1]-[24]. First cosmological constraints from BBN on oscillation parameters were obtained analytically in [8]. Then numerical calculations of the oscillations effect on primordial nucleosynthesis were made in [10]. In these works an account for the depletion of the neutrino number densities due to oscillations was provided, while the neutrino-antineutrino asymmetry and the distortion of the neutrino spectrum were neglected (see also [13]). The importance of the neutrino spectrum distortion for the BBN with oscillations was first noticed for the vacuum oscillation case in ref. [6]. First precise account of both neutrino spectrum distortion and the oscillations generated neutrino-antineutrino asymmetry effects in BBN calculations and cosmological constraints on oscillation parameters were provided in ref. [19].

The problem of active-sterile *non-resonant* neutrino oscillations and the primordial helium-4 production was thoroughly investigated and completed for the model of nonequilibrium oscillations with small mass differences [6, 19] in refs. [21, 22]. There we have analyzed numerically the role of oscillations in BBN using the exact kinetic equations for the neutrino density matrix and nucleon number densities in momentum space. The exact kinetic approach enabled us to reveal the important role of the spectrum distortion and neutrino population depletion. On the other hand, for the nonresonant case it was shown that the lepton asymmetry can be neglected in case initially it was of the order of the baryon one [21], while in case it was greater than 10^{-7} it may considerably influence BBN [22]. Precise constraints on the oscillation parameters by almost an order of magnitude better than the existing ones [10, 13] concerning the neutrino squared mass differences, were obtained due to the exact kinetic approach and selfconsistent account of the evolution of the neutrinos and the nucleons. The analytical fit to the exact constraints [3] is:

$$\delta m^2(\sin^2 2\vartheta)^4 \leq 1.5 \times 10^{-9} \text{ eV}^2 \tag{1}$$

for $\delta m^2 \leq 10^{-7}$ eV^2.

It was noticed in ref. [19], that the *resonant* neutrino oscillations case is a much more complicated one, as far as rapid growth of asymmetry for certain sets of parameters is typical there [8, 16, 18, 19]. First detailed calculations of the BBN with resonant neutrino oscillations accounting for the asymmetry growth were provided in [19]. The phenomenon of the oscillation-generated asymmetry growth was registered there and the calculations of the BBN were provided accounting both for the spectrum distortion and for the neutrino asymmetry dynamical evolution *at each momentum mode*. It was shown that following the behavior of the neutrino-antineutrino asymmetry at each momentum is important, particularly, when the distortion of the neutrino spectrum is considerable. As a result helium-4 contour $Y_p = 0.245$ was obtained and precise constraints on the oscillation parameters were provided. They are better by almost an order of magnitude than the existing ones for the neutrino squared mass differences at large mixing angles.

In ref. [24] the effect of the neutrino-mixing-generated asymmetry was shown to be considerable – up to about 10% relative decrease in helium-4 in comparison with the case with oscillations but without the asymmetry account. Hence, a more profound study of the BBN with resonant oscillations, accounting properly for the important asymmetry effect is necessary. The purpose of the present work is to provide a more detail study of the BBN with resonant neutrino oscillations.

[3] The constraints are derived for primordial helium-4 $Y_p \leq 0.24$. Other iso-helium contours and the corresponding constraints were calculated in refs. [21, 22]

2 Nucleosynthesis with oscillating neutrinos

In the present work we expand and complete the original investigations [19, 24] of the asymmetry effect on primordial production of helium in the model of BBN with resonant neutrino oscillations, for the full parameter space of the nonequilibrium oscillations model [19].

We consider the case of active-sterile neutrino oscillations with small mass differences, namely $\delta m^2 \leq 10^{-7}$ eV^2, described in detail elsewhere [19, 21], where the nonequilibrium effects are stronger and, therefore, it is less studied one till now. According to that model, oscillations proceed effectively after the active neutrino decoupling and till then the sterile neutrinos have not yet thermalized, so that their number density is negligible in comparison with the electron neutrino one. For simplicity we assume mixing just in the electron sector, $\nu_i = U_{il}\ \nu_l$ $(l = e, s)$.

The set of kinetic equations describing simultaneously the evolution of the neutrino and antineutrino density matrix ρ and $\bar{\rho}$ and the evolution of the neutron number density n_n in momentum space reads:

$$\begin{aligned}\frac{\partial \rho(t)}{\partial t} &= H p_\nu \ \frac{\partial \rho(t)}{\partial p_\nu} + \\ &+ i\left[\mathcal{H}_o, \rho(t)\right] + i\sqrt{2} G_F \left(\pm\mathcal{L} - Q/M_W^2\right) N_\gamma \left[\alpha, \rho(t)\right] + \mathrm{O}\left(G_F^2\right), \end{aligned} \quad (2)$$

$$\begin{aligned}&(\partial n_n/\partial t) = H p_n \ \ (\partial n_n/\partial p_n) + \\ &+ \int \mathrm{d}\Omega(e^-, p, \nu) |\mathcal{A}(e^- p \to \nu n)|^2 \left[n_{e^-} n_p (1-\rho_{LL}) - n_n \rho_{LL} (1 - n_{e^-})\right] \\ &- \int \mathrm{d}\Omega(e^+, p, \tilde{\nu}) |\mathcal{A}(e^+ n \to p \tilde{\nu})|^2 \left[n_{e^+} n_n (1-\bar{\rho}_{LL}) - n_p \bar{\rho}_{LL} (1 - n_{e^+})\right] \end{aligned} \quad (3)$$

where $\alpha_{ij} = U^*_{ie} U_{je}$, p_ν is the momentum of electron neutrino, n stands for the number density of the interacting particles, $\mathrm{d}\Omega(i, j, k)$ is a phase space factor and $\mathcal{A}$ is the amplitude of the corresponding process. The sign plus in front of $\mathcal{L}$ corresponds to neutrino ensemble, while minus - to antineutrino ensemble.

The initial condition for the neutrino ensembles in the interaction basis is assumed of the form:

$$\rho = n_\nu^{eq} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

where $n_\nu^{eq} = \exp(-E_\nu/T)/(1+\exp(-E_\nu/T))$.

It corresponds to the standard equilibrium distribution of active electron neutrinos, and an absence of the sterile ones. The initial values for the neutron, proton and electron number densities are their equilibrium values.

The first term in the right hand side of the equations (2) and (3) describes the effect of Universe expansion. The second term in (2) is responsible for neutrino oscillations, the third accounts for forward neutrino scattering off the medium and the last one accounts for the second order interaction effects of neutrinos with the medium. It is important for the nonequilibrium active-sterile neutrino oscillations to provide simultaneous account of the different competing processes, namely: neutrino oscillations, Hubble expansion and weak interaction processes. $\mathcal{H}_o$ is the free neutrino Hamiltonian. The 'nonlocal' term Q arises as an W/Z propagator effect, $Q \sim E_\nu\ T$. $\mathcal{L}$ is proportional to the fermion asymmetry of the plasma and is essentially expressed through the neutrino asymmetries $\mathcal{L} \sim 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}$, where $L_{\mu,\tau} \sim (N_{\mu,\tau} - N_{\bar{\mu},\bar{\tau}})/N_\gamma$ and $L_{\nu_e} \sim \int \mathrm{d}^3p(\rho_{LL} - \bar{\rho}_{LL})/N_\gamma$.

The neutron and proton number densities, used in the kinetic equations for neutrinos eq. (2), were substituted from the numerical calculations of eq. (3). On the other hand, ρ_{LL} and $\bar{\rho}_{LL}$ at each integration step of eq. (3) was taken from the simultaneously performed integration of the set of equations (2). I.e. we have selfconsistently followed the evolution of neutrino ensembles and the nucleons.

We account for the exact kinetics both of the neutrino and the neutron-proton transition, essential for the helium-4 synthesis. Besides, the equations follow neutrino evolution in momentum space, i.e. enabling to account accurately for the neutrino depletion, neutrino energy spectrum distortion and the dynamical evolution of the asymmetry.

Eq. (2) results into a set of coupled nonlinear integro-differential equations with time dependent coefficients for the components of the density matrix of neutrinos: four equations for the components of the neutrino density matrix, and another four for the antineutrino density matrix for each momentum mode. However, due to conservation of the total neutrino number density in the discussed model, the number of the equations can be reduced to 6 equations for each momentum mode of neutrinos and antineutrinos.

The equations were integrated for the characteristic period from the electron neutrino decoupling at 2 MeV till the n/p freeze-out at 0.3 MeV. We have calculated the yields of primordially produced helium-4 for the full range of the model's parameters values, namely for $\sin^2(2\vartheta)$ ranging from 10^{-3} to maximal mixing and 10^{-11} eV^2 $\leq$ δm^2 $\leq$ 10^{-7} eV^2. For smaller mixing parameters the effect on helium-4 was shown to be negligible [19].

Our results are based on hundreds of $\delta m^2 - \vartheta$ combinations. The spectrum distribution we have usually described by 1000 bins. Mind, however, that for some sets of parameters, where rapid growth of asymmetry occurs, even 5000 bins do not give satisfactory good description of the great spectrum distortion and the rapid sign changing behavior of the asymmetry. Fortunately, we have estimated the effect of this numerical uncertainty on the calculated production of helium-4 and found that it is much less than 1% for the full oscillation parameters range.

Therefore, we are not discussing here the asymmetry behavior, but present only the results of our study concerning nucleosynthesis which are trustable. The analysis of the precise asymmetry evolution itself deserves further investigation. We are quite convinced by our studies, that, surely, the real physical behavior of the asymmetry should not be a function of the calculational parameters, such as different error control, step size, et cetera. (See, however, the opposite point of view on that question by Shi in [18]). According to us, such kind of a dependence on the calculational parameters points only to the unsatisfactory accuracy of the numerical calculations or of the calculational methods used. The question is even more complicated, as far as we have estimated that the neutrino evolution equations at resonance have high stiffness. Hence, the usual explicit numerical approach is not applicable for the description of the asymmetry evolution, especially, if the correct account for the spectrum spread of neutrino is provided. To solve the stiff equations numerically, implicit methods should be used. For 1000 bins of the spectrum a system of 6000 equations describing the neutrino density evolution should be solved simultaneously. And this is a hopeless task with our facilities now. We will discuss this question in more detail elsewhere.

3 Results and conclusions

The major effects, of the discussed resonant $\nu_e \leftrightarrow \nu_s$ oscillations with small mass differences on helium-4 production, are due to the depletion of the neutrino number densities, neutrino spectrum distortion and the neutrino asymmetry growth due to oscillations.

(a) Depletion of ν_e population due to oscillations:

As far as oscillations become effective when the number densities of ν_e are much greater than those of ν_s, the oscillations tend to reestablish the statistical equilibrium between different oscillating species. As a result ν_e decreases in comparison to its standard equilibrium value due to oscillations in favor of sterile neutrinos. The depletion of the electron neutrino number densities due to oscillations into sterile ones strongly affects the $n \leftrightarrow p$ reactions rates. It leads to an effective decrease in the weak processes rates, and, hence, to an increase of the freezing temperature of the n/p-ratio and the corresponding overproduction of the primordially produced ^4He.

(b) Distortion of the energy distribution of neutrinos:

Neutrinos with different momenta begin to oscillate at different temperatures and with different amplitudes. First the low energy part of the spectrum is distorted, and later on this distortion concerns neutrinos with higher and higher energies. The effect of the distortion of the energy distribution of neutrinos on helium-4 production is two-fold. On one hand an average decrease of the energy of active neutrinos leads to a decrease of the weak reactions rate, and hence, to an increase in the freezing temperature and the produced helium. On the other hand, there exists an energy threshold for the reaction $\tilde{\nu}_e + p \to n + e^+$. So, in case when the energy of the relatively greater part of neutrinos becomes smaller than that threshold the n/p-freezing ratio decreases leading to a corresponding decrease of the primordially produced helium-4 [25]. The numerical analysis showed that the total effect of the distortion of the energy distribution is an increase in the produced helium.

(c) Neutrino asymmetry:

Neutrino mixing generated asymmetry effect was found to be considerable. (See also ref. [24]). It was proven numerically, that in the case of small mass differences we discussed and naturally small initial asymmetry, the growth of the asymmetry is less than 4 orders of magnitude. Hence, beginning with asymmetries of the order of the baryon one, the asymmetry does not grow enough to influence *directly* the kinetics of the $n-p$ transitions. Consequently, the apparently great asymmetry effect (as illustrated in Fig. 2) is totally due to the *indirect* effects of the asymmetry on BBN. The maximal asymmetry effect is around 10% 'underproduction' of Y_p in comparison with the case of BBN with oscillations but without the asymmetry account.

The total effect of oscillations, with the complete account of the asymmetry effects, is still overproduction of helium-4, in comparison to the standard value, although considerably smaller at small mixing angles than in the calculations neglecting asymmetry. Therefore, nucleosynthesis constraints on the mixing parameters of neutrino are alleviated considerably due to the asymmetry effect.

From the numerical integration for the full range of oscillation parameters we have obtained the primordial helium yields $Y_p(\delta m^2, \vartheta)$. Some of the iso-helium contours calculated in the discussed model of cosmological nucleosynthesis with resonant neutrino oscillations are presented on the plane $\delta m^2 - \vartheta$ in Fig. 1.

At present the primordial helium values extracted from observations differ considerably [26]. Therefore, we consider it useful to provide the exact calculations for various iso-helium contours up to 0.26. Knowing more precisely the primordial helium-4 value from observations, it will be possible to obtain the excluded region of the mixing parameters using the results of this survey. For example, assuming the 'low' observational value of primordial ^4He $Y_p \cong 0.234$ [26], the cosmologically excluded region for the oscillation parameters is situated on the plane $\delta m^2 - \vartheta$ to the right of the $Y_p = 0.245$ curve, which gives 5% overproduction of helium in comparison with this observational value.

In Fig. 2 a comparison between the curves, corresponding to helium abundance $Y_p = 0.24$, obtained in the present work and in previous works [10, 13], analyzing the resonant active-sterile neutrino oscillations, is presented. In [10] the excluded regions for the neutrino mixing parameters were obtained from the requirement that the neutrino types should be less than 3.4: $N_\nu < 3.4$. The depletion effect was considered, while the neutrino-antineutrino asymmetry was neglected, and the distortion of the neutrino spectrum was not studied, instead the kinetic equations for neutrino mean number densities were used.

The dashed curve, presenting our results, in case the asymmetry effect was neglected, is in a good accordance with the results of Enqvist et al. [10], where asymmetry was neglected. The difference between the two curves shows explicitly the effect of the proper account of the neutrino spectrum spread and spectrum distortion, which was provided in our work. On the other hand, the difference between our curves, the solid and the dashed one, presents the net asymmetry effect.

The results of [13] differ both from the ones of ref. [10] and from our results. We consider them not correct. Our conclusion is not only based on the discrepancy between these results and those of other studies, but on the very fact that they are not consistent even between themselves concerning resonant and nonresonant case. As is well known from the analytical formulae the results for the resonant case $\delta m^2 < 0$ coincide with those for the nonresonant one $\delta m^2 > 0$ at maximal mixing. This fact is illustrated in Fig. 3 of resonant and nonresonant oscillations for all studies, except ref. [13].

As is seen from the iso-helium contours for $Y_p = 0.24$, for *large mixing angles* we exclude mass differences $\delta m^2 \geq 8.2 \times 10^{-10}$ eV^2, which is an order of magnitude stronger constraint than the previously existing. This more stringent constraint for mass differences, obtained in our work for the region of large mixing angles is due to the more accurate kinetic approach we have used and to the precise account of neutrino depletion and energy distortion. On the other hand, at *small mixing angles* the account of the oscillations generated asymmetry leads to an alleviation of the BBN constraints in comparison with the previous works [10, 13]. It is easy to understand, as far as the asymmetry growth results in suppression of oscillations and, hence, less strongly pronounced overproduction of helium-4 due to oscillations than in the case without the asymmetry account.

In conclusion, we have shown that both the spectrum distortion and neutrino mixing generated asymmetry should be accounted for properly in models of BBN with oscillations, as far as their effect is considerable. We have calculated different iso-helium contours for the resonant case of neutrino oscillations with small mass differences. The cosmological constraints obtained are better by an order of magnitude than the existing ones due to the exact kinetic approach both to the neutrino evolution and to the nucleons freeze-out.

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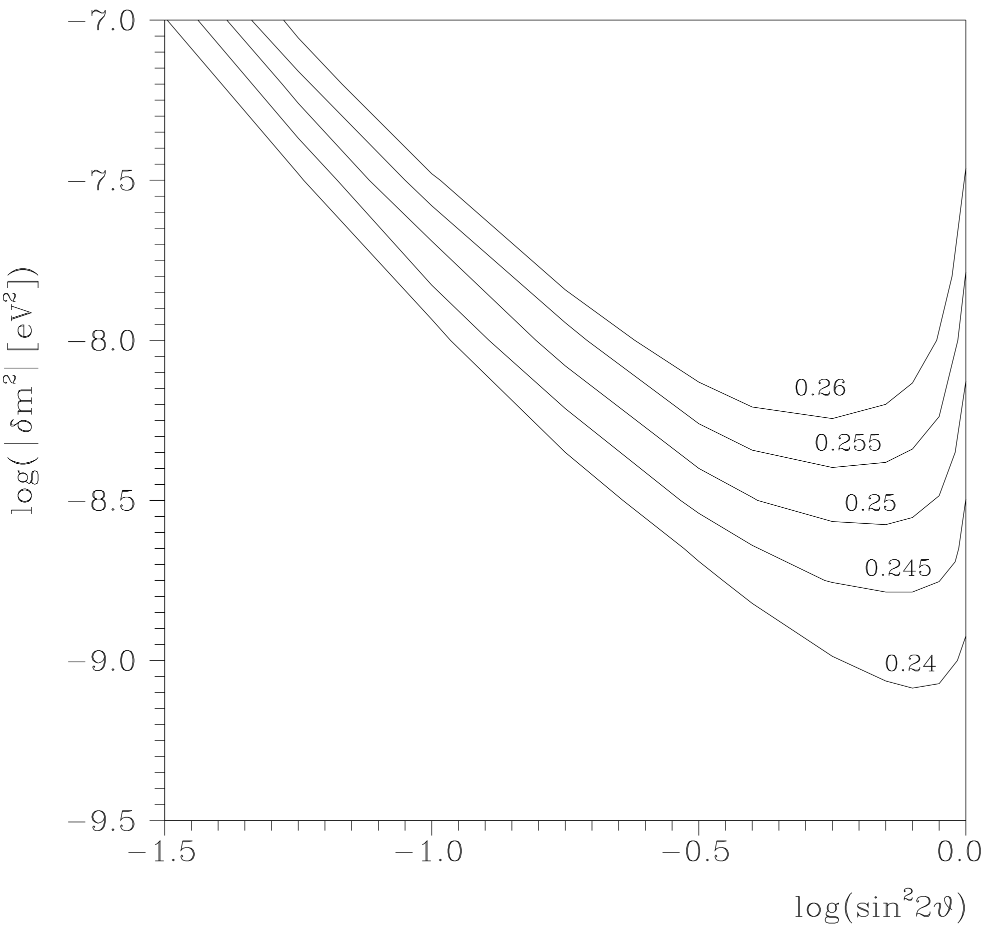
Figure captions

Figure 1. On the $\delta m^2 - \vartheta$ plane iso-helium-4 contours $Y_p = 0.24$, 0.245, 0.25, 0.255 and 0.26, calculated in the discussed model of BBN with active-sterile resonant neutrino oscillations are shown. For fixed primordial helium-4 value, the area to the left of the corresponding curve gives the allowed region of the oscillation parameters.

Figure 2. In the figure a comparison between the results concerning primordial helium-4 production, obtained in the present work and previous works [10, 13], is presented. The dashed curve shows our results in case without asymmetry effect account. It is in a good accordance with the results of Enqvist et al. [10], where asymmetry was neglected. The difference between the two curves shows explicitly the effect of the proper account of the spectrum spread of neutrino, which was provided in our work. On the other hand, the difference between our curves, the solid and the dashed one presents the net asymmetry effect. The artistic curve of Shi et al. [13] is obviously inconsistent with the results of other works and we will leave it without a comment.

Figure 3. Combined iso-helium contours $Y_p = 0.24$, for the resonant oscillations, $\delta m^2 < 0$, and the nonresonant ones, $\delta m^2 > 0$, calculated in previous studies [10, 13, 21] and in this work, are presented. The discontinuity of the curve of Shi et al. [13] reveals the discrepancy between their own results for the resonant and nonresonant case.



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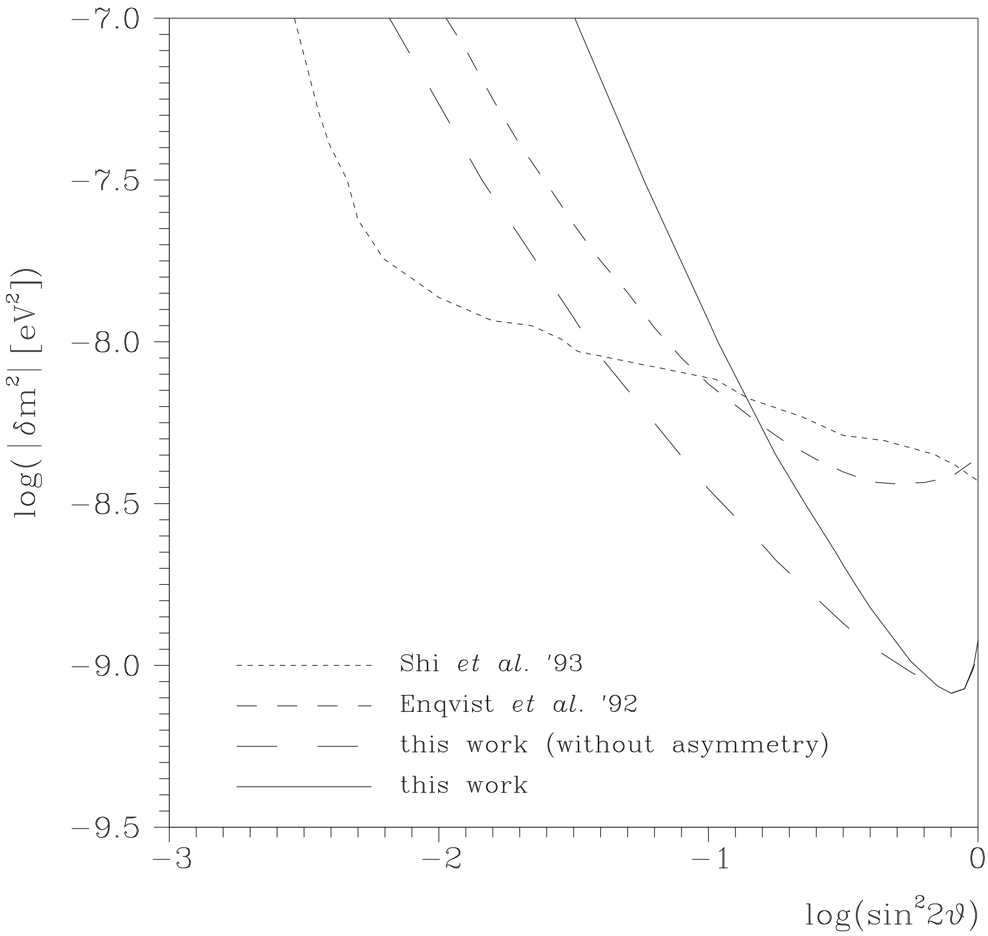

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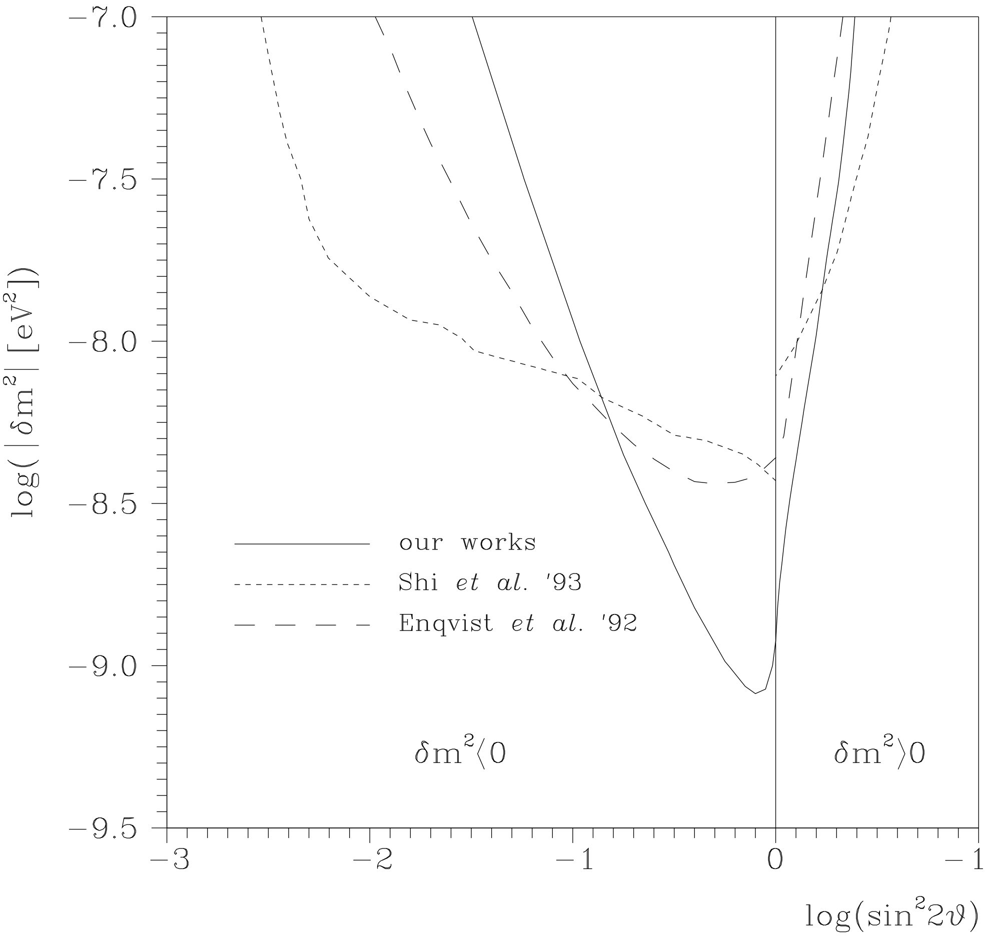

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